



# Vacuum energy with mass generation and Higgs bosons

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## ABSTRACT

We discuss the Higgs mass and cosmological constant hierarchy puzzles with emphasis on the interplay of Poincaré invariance, mass generation and renormalization group invariance. A plausible explanation involves an emergent Standard Model with the cosmological constant scale suppressed by power of the large scale of emergence. In this scenario the cosmological constant scale and neutrino masses should be of similar size.

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## 1. Introduction

Particle Physics comes with two mysterious hierarchies of scales. The mass of the Higgs boson discovered at CERN is about 125 GeV [1,2]. Theoretically the Higgs mass squared comes with a quadratically divergent counterterm which naively would push its value towards the Planck scale. The cosmological constant or vacuum energy density which drives the accelerating expansion of the Universe is characterized by a scale 0.002 eV [3], very much less than the Higgs mass, QCD and Planck scales. It is an open question whether these two puzzles might be connected. Here we discuss these scale hierarchies with focus on the interplay of Poincaré invariance, mass generation and renormalization group invariance. We argue that a plausible explanation involves an emergent Standard Model [4,5] and the measured cosmological constant scale associated with higher dimensional terms in the action, suppressed by power of the large emergence scale. In this scenario it is natural that the cosmological constant scale and neutrino masses should be of similar size.

In Section 2 we next discuss the scale hierarchies associated with ultraviolet divergences and renormalization, e.g. the Higgs mass counterterm and zero-point energies induced by quantization. Both the Higgs mass counterterm and net zero-point energy contribution to the cosmological constant are related to the relative contributions of bosons and fermions, including possible ex-

tra Higgs bosons. Consequences for new particles at LHC energies are discussed. Section 3 discusses the effect of running masses and couplings, how the Standard Model parameters are linked to physics in the ultraviolet. Finally, in Section 4 we consider the cosmological constant and explain why a tiny non-zero value fits with possible emergent gauge symmetry. Large zero-point energy contributions to the cosmological constant are renormalization scale dependent and decouple. With emergence, contributions from potentials linked to the Higgs and QCD condensates are cancelled at mass dimension four in the action.

## 2. Scale hierarchies and renormalization

In the Standard Model the masses of the W and Z gauge bosons and charged fermions come from coupling to the Higgs boson with a finite Higgs vacuum expectation value, vev. The renormalized Higgs mass squared comes with the divergent counterterm

$$m_h^2 \text{ bare} = m_h^2 \text{ ren} + \delta m_h^2 \quad (1)$$

where

$$\delta m_h^2 = \frac{K^2}{16\pi^2} \frac{6}{v^2} \left( m_h^2 + m_Z^2 + 2m_W^2 - 4m_t^2 \right) \quad (2)$$

relates the renormalized and bare Higgs mass. Here K is an ultraviolet scale characterizing the limit to where the Standard Model should work, v is the Higgs vev and the  $m_i$  are the Higgs, Z, W and top quark masses. We neglect contributions from lighter mass quarks.

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Boson and fermion contributions enter with different signs and come with different renormalization scale dependence. The renormalized and bare masses would coincide with no hierarchy puzzle if

$$2m_W^2 + m_Z^2 + m_h^2 = 4m_t^2. \quad (3)$$

This equation is the Veltman condition [6]. It implies a collective cancellation between bosons and fermions. Taking the pole masses for the W, Z and top quark (80, 91 and 173 GeV) would require a Higgs mass of 314 GeV, much above the measured value. Running masses are discussed below.

A similar discussion follows for the vacuum zero-point energies, ZPEs, of quantum field theory which are induced by quantization [7] and important in the cosmological constant [8] together with potentials associated with vacuum condensates. We work in flat space-time. Zero-point energies come with ultraviolet divergence requiring regularization and renormalization,

$$\rho_{\text{zpe}} = \frac{1}{2} \sum \{\hbar\omega\} = \frac{1}{2} \hbar \sum_{\text{particles}} g_i \int_0^{k_{\text{max}}} \frac{d^3k}{(2\pi)^3} \sqrt{k^2 + m^2}. \quad (4)$$

Here  $\frac{1}{2}\{\hbar\omega\}$  denotes the eigenvalues of the free Hamiltonian and  $\omega = \sqrt{k^2 + m^2}$  where  $k$  is the wavenumber and  $m$  is the particle mass;  $g_i = (-1)^{2j}(2j+1)f$  is the degeneracy factor for a particle  $i$  of spin  $j$ , with  $g_i > 0$  for bosons and  $g_i < 0$  for fermions. The minus sign follows from the Pauli exclusion principle and the anti-commutator relations for fermions. The factor  $f$  is 1 for bosons, 2 for each charged lepton and 6 for each flavour of quark (2 charge factors for the quark and antiquark, each with 3 colours).

It is important to choose a Lorentz covariant regularization to ensure that the renormalized zero-point energy satisfies the correct vacuum equation of state. Dimensional regularization with minimal subtraction,  $\overline{\text{MS}}$ , is a good regularization. One finds

$$\rho_{\text{zpe}} = -p_{\text{zpe}} = -\hbar g_i \frac{m^4}{64\pi^2} \left[ \frac{2}{\epsilon} + \frac{3}{2} - \gamma - \ln\left(\frac{m^2}{4\pi\mu^2}\right) \right] + \dots \quad (5)$$

from particles with mass  $m$  [9]. Here  $p_{\text{zpe}}$  is the pressure,  $D = 4 - \epsilon$  the number of dimensions,  $\mu$  the renormalization scale and  $\gamma$  is Euler's constant. If one instead uses a brute force cut-off on the divergent integral, the leading term in the ZPE proportional to  $k_{\text{max}}^4$  obeys the radiation equation of state  $\rho = p/3$ .

Equation (5) means that the ZPE vanishes for massless particles, e.g. photons. For Standard Model particles it is induced by the Higgs mechanism. Boson and fermion contributions to the net zero-point energy come with different signs. This led Pauli to suggest a collective cancellation of the ZPE [10], much like the Veltman condition for the Higgs mass squared. If we take the ZPEs as physical, then this Pauli constraint for cancelling the ZPE gives new constraints on possible extra particles [10,11]

$$\sum_i g_i m_i^4 = 0$$

$$\sum_i g_i m_i^4 \ln m_i^2 = 0. \quad (6)$$

For the Standard Model with the physical W, Z and top-quark masses, these two equations would need a Higgs mass of about 319 GeV and 311 GeV respectively, close to the Veltman value of 314 GeV. With the Standard Model particle masses, the ZPE is negative and fermions dominated. We need some extra bosons if we want this to work. (Contributions to the total vacuum energy

from the Higgs potential and QCD condensates are discussed in Section 4 below.)

Obvious first candidates are 2 Higgs Doublet Models [12]. These are a simple extension of the Standard Model. One introduces a second Higgs doublet. There are 5 Higgs bosons, two neutral scalars  $h$  and  $H$ , one pseudoscalar  $A$  and two charged Higgs states  $H^\pm$ . Since the 125 GeV Higgs-like scalar discovered at CERN in 2012 [1,2] has so far showed no departure from Standard Model predictions, it must be assumed in any model with extra Higgs states that one of the neutral scalars  $h$  is a lot like the Standard Model Higgs.

The possible extra Higgs states are looked for in direct searches [13,14]. The parameter space is constrained with lower bounds on the masses from global electroweak fits [15] and rare B-decay processes [16,17]. Different model scenarios depend on the fermion to Higgs couplings. The most constrained are Type II models with  $600 \text{ GeV} < m_{H^\pm}$ ,  $530 \text{ GeV} < m_A$  and  $400 \text{ GeV} < m_H$ . Here one doublet couples to up type quarks and one to down type quarks and leptons. Others are the type I fermiophobic model where all fermions couple to just one doublet, lepton specific (one doublet to quarks and one to leptons) and flipped (same as type II except leptons couple to the doublet with up type quarks). There are also inert models where only one doublet acquires a vev and couples to fermions. These models are less well constrained. For the Veltman condition extended to 2HDMs, a favoured benchmark point is quoted in the Type II model with  $m_H \sim 830 \text{ GeV}$  and  $m_A, m_{H^\pm} \sim 650 \text{ GeV}$  [18]. Within the mass constraints quoted for the Type II models, bosons win! We would need also extra fermions in the energy range of the LHC to cancel the Pauli condition with the allowed masses.

### 3. Running masses

We next turn to running masses.

Both the Veltman and Pauli constraints are evaluated from loop diagrams so the masses which appear there are really renormalization group, RG, scale dependent. Boson and fermion contributions enter with different signs and evolve differently under RG evolution which means they have a chance to cross zero deep in the ultraviolet. With the particle masses and couplings measured at the LHC, the Standard Model works as a consistent theory up to the Planck scale. One finds that the electroweak vacuum sits very close to the border of stable and metastable suggesting possible new critical phenomena in the ultraviolet, within 1.3 standard deviations of being stable on relating the top quark Monte-Carlo and pole masses if we take just the Standard Model with no coupling to undiscovered new particles [19]. The question of vacuum stability depends on whether the Higgs self-coupling crosses zero or not deep in the ultraviolet and involves a delicate balance of Standard Model parameters. The Higgs and other particle masses might be determined by physics close to the Planck scale.

The scale of Veltman crossing is calculation dependent. Values reported are  $10^{16} \text{ GeV}$  with a stable vacuum [4], about  $10^{20} \text{ GeV}$  [20] and much above the Planck scale of  $1.2 \times 10^{19} \text{ GeV}$  [21,22] with a metastable vacuum. With the Standard Model evolution code [23] crossing is found at the Planck scale with a Higgs mass about 150 GeV, and not below with the measured mass of 125 GeV. (The 125 GeV mass is close to the minimum needed for vacuum stability.)

### 4. The cosmological constant

Experimentally, vacuum energy becomes important through the cosmological constant  $\Lambda$ . This measures the vacuum energy density  $\rho_{\text{vac}} = \Lambda/(8\pi G)$  where  $G$  is Newton's constant. The cosmolog-

ical constant appears on the right-hand side of Einstein's equations of General Relativity

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\frac{8\pi G}{c^2}T_{\mu\nu} + \Lambda g_{\mu\nu}. \quad (7)$$

Here  $R_{\mu\nu}$  is the Ricci tensor,  $R$  is the Ricci scalar and  $T_{\mu\nu}$  is the energy-momentum tensor for excitations above the vacuum. The cosmological constant receives contributions from the ZPEs, any (dynamically generated) potential in the vacuum, e.g. associated with the Higgs and QCD condensates, and a renormalized version of the bare gravitational term  $\rho_\Lambda$  [8,24].

The net vacuum energy density after turning on particle interactions,

$$\rho_{\text{vac}} = \rho_{\text{ZPE}} + \rho_{\text{potential}} + \rho_\Lambda, \quad (8)$$

is renormalization scale invariant. It drives the accelerating expansion of the Universe and is independent of how a theoretician might choose to calculate it,

$$\frac{d}{d\mu^2}\rho_{\text{vac}} = 0. \quad (9)$$

Before we couple to gravity only energy differences have physical meaning, which then allows us to cancel the zero-point energy contribution by normal ordering before consideration of vacuum potentials induced by spontaneous symmetry breaking. The Casimir force which is sometimes claimed as experimental evidence for ZPEs can also be calculated without reference to ZPEs [25]. Calculation of the Casimir effect involves Feynman graphs with external lines whereas the ZPE does not, just closed loops.

The ZPE contributions in Eq. (5) are scale dependent both through explicit  $\mu^2$  dependence and through the running masses. For QCD, the degrees of freedom depend on the resolution. Deep in the ultraviolet one has asymptotic freedom. For massless quarks, the ZPE vanishes. Quark-gluon interactions are chiral symmetric at these scales. In the infrared confinement and dynamical chiral symmetry breaking take over: the degrees of freedom are protons, neutrons, pions, nucleon resonances... If energy conservation held for the ZPE (plus QCD potential terms) one would find a constraint condition on the hadron spectrum from summing over hadronic ZPE contributions. The Higgs potential is RG scale dependent through the scale dependence of the Higgs mass and Higgs self-coupling, which determines the stability of the electroweak vacuum. QCD quark and gluon condensates also enter. Renormalization scale dependence cancels to give the scale invariant  $\rho_{\text{vac}}$ . The important question is whether, after we sum over the ZPEs, potentials in the vacuum and the  $\rho_\Lambda$  contribution in Eq. (8), there is anything left over. How big is the remaining  $\rho_{\text{vac}}$ ?

A clue may be the curious result that with a finite cosmological constant Einstein's equations have no solution where  $g_{\mu\nu}$  is the constant Minkowski metric [8] (A non-vanishing  $\rho_{\text{vac}}$  acts as a gravitational source which generates a dynamical space-time, with accelerating expansion for positive  $\rho_{\text{vac}}$ ). For the vacuum with net constant field,  $\rho_{\text{vac}} \neq 0$ , space-time translational invariance is broken without extra fine tuning. This, in turn, challenges the flat space-time with covariance assumed in Eq. (5).

So far we have treated the Standard Model as fundamental. Interactions are determined by gauge symmetries. The theory is covariant and renormalizable and described by an action with terms of dimension four or less.

The Standard Model and its gauge symmetries might be emergent [4,5,26–29]. For example, Standard Model particles could be the long-range, collective excitations of a statistical system near to its critical point that resides close to the Planck scale [4]. With the Standard Model as an effective theory emerging in the infrared,

low-energy global symmetries can be broken through additional higher dimensional terms, suppressed by powers of a large ultraviolet mass scale [5,29]. Gauge symmetries would be exact, modulo the Higgs coupling, within the effective theory. Suppose the vacuum including condensates with finite vevs is translational invariant and flat space-time is consistent at dimension four, just as suggested by the success of the Standard Model. Then the RG invariant scales  $\Lambda_{\text{QCD}}$  and electroweak  $\Lambda_{\text{EW}}$  might enter the cosmological constant with the scale of the leading term suppressed by  $\Lambda_{\text{EW}}/M$ , where  $M$  is the scale of emergence (that is,  $\rho_{\text{vac}} \sim (\Lambda_{\text{EW}}^2/M)^4$  with one factor of  $\Lambda_{\text{EW}}^2/M$  for each dimension of space-time). This scenario, if manifest in nature, would explain why the cosmological constant scale 0.002 eV is similar to what we expect for the neutrino masses [30], themselves linked to a dimension five operator with  $m_\nu \sim \Lambda_{\text{EW}}^2/M$  and Majorana neutrinos [31]. The cosmological constant would vanish at dimension four. That is,  $\rho_{\text{vac}} = 0$  follows as a renormalization condition at dimension four set by space-time translational invariance, even in the presence of Higgs and QCD vacuum condensates. The precision of global symmetries in our experiments, e.g. lepton and baryon number conservation, tells us that the scale of emergence should be deep in the ultraviolet, much above the Higgs and other Standard Model particle masses.

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